

# Monte Carlo Tools for Jet Quenching

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**Abstract.** A thorough understanding of jet quenching on the basis of multi-particle final states and jet observables requires new theoretical tools. This talk summarises the status and prospects of the theoretical description of jet quenching in terms of Monte Carlo generators.

## 1. Introduction

The motivations for studying jet observables instead of single-inclusive observables are manifold and shall not be discussed extensively here. Instead, only two important points will be mentioned. Firstly, it has turned out that single-inclusive observables do not fully constrain the analytical models for partonic energy loss. The nuclear modification factor for instance is described by all models equally well albeit with very different transport coefficients[1]. The benefit of studying sub-leading fragments and jet observables is that these are much more discriminating. The downside is that they are not well modelled by the existing analytical calculations and new theoretical tools are needed.

Secondly, on the experimental side there is need for reliable tools that allow to disentangle jets from the background. This requires a quantitative understanding of jet areas, background and background fluctuations[2] and the response of jet finding algorithms to quenched jets[3]. Further complications arise from the fact that the implicit assumption that jets and background are uncorrelated is strictly speaking not justified. It is to be expected on general grounds that there is a backreaction of the jets to the medium, but this is very difficult to quantify with the currently available tools. It is, however, clear that a quantitative understanding of jets, background and their correlations requires running jet finders on the theory predictions.

In particle physics Monte Carlo (MC) event generators have emerged as extremely powerful and versatile tools linking theory and experiment. It is the aim of this talk to make an assessment of the status of MC models for jet quenching by highlighting conceptual issues and to what extent they are solved or solvable using MC tools. It focusses on dedicated jet quenching MC models and does not discuss parton cascade codes. The latter should in principle also be capable of describing jet quenching, but

as the approach and focus are quite different from specialised jet quenching MC codes they will not be discussed here.

## 2. Jets in p+p

Jet evolution in vacuum arises due to collinear divergences in real emission matrix elements. The singularity structure is universal, leading to a factorisation in the collinear region. The differential cross section for a process with  $n + 1$  partons can thus be written as the product of the differential cross section for the process with  $n$  partons, the single particle phase space and the differential radiation probability characterised by the Altarelli-Parisi splitting function:

$$d\sigma_{n+1} \approx d\sigma_n \frac{dt}{t} \frac{d\phi}{2\pi} dz \frac{\alpha_s}{2\pi} \mathcal{P}(z) \quad (1)$$

The variable  $t$  quantifies the hardness of the splitting and can be the transverse momentum squared  $k_\perp^2$ , the virtuality  $Q^2$  or the radiation angle squared  $\vartheta^2$  – all these choices are to leading logarithmic accuracy equivalent. The regularisation of the  $1/t$ -singularity gives rise to large logarithms that can compensate the smallness of the strong coupling  $\alpha_s$  and thus have to be resummed to all orders. This proceeds via a DGLAP evolution equation, which for only one parton species can be written as

$$t \frac{\partial}{\partial t} f(x, t) = \int_x^1 dz \frac{\alpha_s}{2\pi} \mathcal{P}(z) \left[ \frac{1}{z} f(x/z, t) - f(x, t) \right]. \quad (2)$$

Here,  $f(x, t)$  is the parton density at energy fraction  $x$  and scale  $t$ . The change in  $f(x, t)$  when increasing the scale is given by an increase due to splittings of partons at higher energy fraction  $x$  and a decrease due to partons at  $x$  that split to populate smaller energies. This equation can be integrated with the help of the Sudakov form factor defined as

$$\mathcal{S}(t_1, t_2) = \exp \left\{ - \int_{t_1}^{t_2} \frac{dt}{t} \int dz \frac{\alpha_s}{2\pi} \mathcal{P}(z) \right\}. \quad (3)$$

The integrated evolution equation then becomes

$$f(x, t) = \mathcal{S}(t_0, t) f(x, t_0) + \int_{t_0}^t \frac{dt'}{t'} \mathcal{S}(t', t) \int \frac{dz}{z} \frac{\alpha_s}{2\pi} \mathcal{P}(z) f(x/z, t'), \quad (4)$$

indicating that  $\mathcal{S}(t_0, t)$  can be interpreted as the probability for having no emission between  $t$  and  $t_0$ . Here,  $t_0$  is the infra-red cut-off scale, at which the dynamics becomes dominated by non-perturbative physics. As it regularises a soft and collinear divergence it is important that the observables that are to be calculated are insensitive to the exact value of the cut-off, i.e. are infra-red and collinear safe.

The Sudakov form factor is the basis for an iterative Monte Carlo (MC) algorithm that generates the radiative corrections to any given hard process and is called the

parton shower. It resums real emissions to all orders to leading logarithmic accuracy and via unitarity also includes the corresponding virtual corrections.

The elements of Monte Carlo event generators that are relevant for jet production and evolution are matrix elements, parton showers and the hadronisation. The matrix elements describe hard scattering processes and are calculated at fixed order in perturbation theory. Considerable effort has been made in the last years to provide the matrix elements for all phenomenologically relevant processes at next-to-leading order in the MC generators and to interface them with the parton showers. The initial and final state parton showers generate radiative corrections to the matrix elements by resumming collinear or soft and collinear logarithms to leading logarithmic approximation  $\ddagger$ . These parts of the event generators (i.e. matrix elements and parton showers) are nothing but a faithful representation of perturbation theory with controlled systematic accuracy. In contrast to this, the hadronisation is governed by non-perturbative physics and is not accessible with perturbative methods. It is instead simulated with the help of phenomenological models, the most commonly used and best tested models are the Lund string model and the cluster hadronisation. The model parameters are tuned to data (mainly from LEP) and assumed to be universal.

### 3. Jets in A+A

In heavy ion collisions the hard matrix elements remain unchanged, due to the high scale they involve. They thus happen on very short time and length scales early in the reaction and are not affected by the formation of a system of considerable density. The final state parton shower, on the other hand, evolves over a considerably longer time scale and is most likely modified by the formation of a dense and hot medium. There is, however, no general theory for jet evolution in a dense medium, but only results for special cases (for instance the single gluon radiation spectrum in the eikonal limit). The initial state parton shower was found to be unmodified by the RHIC experiments. The hadronisation, finally, is likely to experience modifications, but due to its non-perturbative nature there is only very little theoretical guidance.

The parton shower as introduced in section 2 is formulated in momentum space. The theoretical description of jet evolution in a medium requires a simultaneous formulation in momentum and configuration space. This can be achieved by estimating the time scale for gluon radiation from the uncertainty principle:

$$\tau_{\text{vac}} \approx \frac{\omega}{k_{\perp}^2} \approx \frac{1}{\sqrt{t}} \cdot \frac{E}{\sqrt{t}} \quad (5)$$

The first expression is simply the inverse transverse energy of the radiated gluon, while the second is the lifetime  $1/\sqrt{t}$  of an unstable state decaying into two partons characterised by a scale  $t$  boosted to the laboratory frame. The two arguments are parametrically equivalent. Apparently the formation or decoherence of energetic

$\ddagger$  In fact, modern parton showers also contain certain next-to-leading logarithmic terms.

fragments in the vacuum parton shower is delayed by time dilation to times typically larger than the medium length.

In the case of medium induced radiation the transverse momentum comes predominantly from scattering in the medium:  $k_{\perp}^2 = \hat{q}\tau_{\text{med}}$ . One thus finds for the lifetime of medium induced radiation

$$\tau_{\text{med}} \approx \frac{\omega}{k_{\perp}^2} \approx \frac{\omega}{\hat{q}\tau_{\text{med}}} \Rightarrow \tau_{\text{med}} \approx \sqrt{\frac{\omega}{\hat{q}}} \quad (6)$$

This means that soft emissions decohere first and at large angles.

At face value these qualitative estimates indicate that jets emerging from heavy ion collisions should have a hard core that fragments in vacuum accompanied by soft medium induced radiation at large angles[4]. This is in qualitative agreement with early jet measurements by ATLAS[5] and CMS[6] and can be further clarified and quantified by measurements of intra-jet distributions (for instance fragmentation functions).

The currently available MC models for jet quenching follow rather different approaches. They shall only be briefly mentioned here, for details the reader is referred to the original publications.

**HIJING** introduces a medium induced parton splitting process, collisional energy loss is neglected[7, 8]

**HYDJET++/PYQUEN** simulates radiative energy loss by sampling a BDMPS gluon spectrum and includes perturbative elastic scattering[10, 9]

**JEWEL** aims for a unified description of all scattering processes in terms of matrix elements and parton showers (work in progress)[11, 12]

**Q-PYTHIA/Q-HERWIG** adds a term derived from BDMPS to the splitting function and thus contains no elastic scattering[13, 14]

**YaJEM** assumes that medium interactions increase the virtuality of partons in the parton shower (leading to enhanced radiation) and can subtract energy and momentum to simulated collisional energy loss[15, 16]

**MARTINI** is based on the AMY transition rates and also contains a transition rate for elastic scattering[17, 18]

Going from single-inclusive to jet observables raises conceptual issues, that will be discussed in the following.

### 3.1. Non-eikonal kinematics

The analytical calculations of radiative energy loss operate in the eikonal approximation where

$$E \gg \omega \gg k_{\perp}, q_{\perp} \gg \Lambda_{\text{QCD}}. \quad (7)$$

Consequently, energy and momentum are not conserved and the scattering centres are recoilless, i.e. there is no collisional energy loss. In contrast to this, phenomenology

at RHIC and LHC requires that all quantities in the above equation can be of the same order, but are subject to energy-momentum conservation. There are thus large uncertainties in the predictions of analytical calculations due to kinematical ambiguities.

In general, i.e. non-eikonal, kinematics phase space restrictions due to energy-momentum conservation have to be taken into account. This is not necessarily a small effect, but can have rather dramatic effects on the gluon spectrum. Furthermore, the scattering centres become dynamical and recoil against the jet giving rise to collisional energy loss (also in inelastic interactions) and induced radiation off the scattering centre. This already hints at an additional complication, namely that elastic and inelastic scatterings cannot be unambiguously separated from each other. They are two possible outcomes (depending on a definition) of the same process rather than different processes. There is also no clear separation between vacuum and medium induced radiation any more.

Incorporating exact energy-momentum conservation is for MC models typically straight forward. The ambiguity between elastic and inelastic scattering, on the other hand, requires a unified description of both processes, which is currently under development. Dynamical scattering centres are problematic for models based on effective descriptions for the medium interactions but are less challenging for models incorporating microscopical interaction models. Therefore, collisional energy loss is typically either neglected or added as a separate process. Radiation off the scattering centres requires model dependent assumptions and has not been included yet, but first steps are being taken. Concerning the ambiguity between vacuum and medium induced radiation the models either build on an unified description and don't distinguish at all between the two, or they assume a complete factorisation where all vacuum radiation happens first and the medium induced emissions follow afterwards.

### 3.2. Multiple gluon emission and LPM-effect

The analytical models only compute single gluon radiation, which is then iterated probabilistically. This leads to complications due to the non-conservation of energy and momentum in the eikonal limit. Concerning the interplay between multiple gluon emissions first theoretical progress was made recently through the calculation of gluon radiation off a colour dipole[19, 20]. The results indicate, that interference between emissions off the two legs occurs only in restricted regions of phase space so that the dominant process is independent radiation off both legs of the dipole[20].

Another important effect is that radiated gluons themselves radiate requiring a democratic treatment of all partons. This can have significant effects in particular on  $k_{\perp}$ - and angular distributions. This also means that the energy loss is not a meaningful quantity any more and that rather the entire fragmentation pattern has to be considered. Energy-momentum conservation is already important for single gluon radiation, for the description of multi-parton final states it is crucial.

These considerations allow for the conclusion, that theories without democratic radiation are not suitable for jet phenomenology.

Treating the non-Abelian Landau-Pomeranchuk-Migdal-effect, which is a quantum-mechanical interference, is notoriously difficult for MC models. Therefore, most of them use effective descriptions of the single gluon radiation process. However, in [12, 21] a local and probabilistic formulation of the LPM-effect was derived, that will also be included in future MC models.

The MC models also iterate single gluon radiation, which always involves model dependent assumptions. The common assumption that all partons radiate independently has received support from the recent results on gluon radiation off colour dipoles. Democratic treatment is by construction easily achieved (except for the scattering centre).

### 3.3. $k_{\perp}$ -broadening

Understanding  $k_{\perp}$ -broadening is important as it affects the response of jet finders to quenched jets. In the analytical models it is governed by Brownian motion in the transverse space. It is, however, sensitive to energy-momentum conservation, democratic multiple gluon radiation and contamination by energetic recoils.

The assumptions in the MC models vary from collinear gluon emission to parton shower kinematics and generally leave room for improvements of the microscopic dynamics.

### 3.4. Recoils, medium modelling and background

The propagation of highly energetic jets leads to modifications of the medium in the vicinity of the jets, which is likely to survive into the hadronic stage. A quantitative understanding of jet-induced modifications of the medium is thus important for the experimental background subtraction. But it is also interesting in its own right, as it for instance gives access to the interplay between weakly and strongly coupled regimes. A satisfactory level of understanding of the jet backreaction probably requires a unified description of jet and medium evolution. While this is still some way to go first attempts to characterise the reaction of the medium to jets have been made from the jet quenching side by tracking the recoiling scattering centres [11] and from the medium modelling side by solving hydrodynamics with source terms that describe the energy and momentum deposition of a jet [22].

The fact that most MC models use hydrodynamic calculations as model for the medium makes it difficult for them to quantify the backreaction. A conversion of hydrodynamical results into a population of scattering centres is possible, but involves model dependent assumptions. Even then, the information about the distortion due to the passage of a jet cannot be propagated back to the hydrodynamical calculation. Event-by-event hydrodynamics is a development that may allow for a simultaneous

solution in the future, but the problem of having two vastly different approaches for the jet and the medium evolution remains. This is partly a problem of very different scales, as the jets are described in perturbation theory requiring a high scale, whereas the bulk medium evolution is governed by non-perturbative processes at much lower scales. Consequently, also the language is vastly different with the jets being formulated in terms of individual partons while the medium is usually treated using continuum dynamics. Bringing both regimes together is thus far from trivial. Parton cascades that are built on a partonic language also for the medium have so far not been able to consistently include the parton shower.

### 3.5. Hadronisation

Medium modifications to the hadronisation phase are likely and raise conceptually both interesting and difficult questions. Due to the non-perturbative nature of the problem there is at best very little theoretical guidance. Therefore, it is commonly assumed that hadronisation happens at late times and therefore in vacuum. This is a reasonable assumption for energetic fragments due to the large boost factors, but not necessarily for soft and semi-hard fragments. Even if one assumes hadronisation in vacuum there are complications due to the medium: The hadronisation is sensitive to the colour topology of the event, but strong interactions in a medium inevitably alter the colour configuration. Furthermore, it is unclear how jet and medium hadronisation interplay, in particular in regions of phase space where soft fragments of the jets overlap with relatively hard fluctuations or recoils from the medium. This leads to potentially large uncertainties even in a factorised approach.

MC models assume – like most other models – that hadronisation happens in vacuum. Some allow for modifications of the colour topology, but with the exception of Q-HERWIG they all rely on the Lund string model. As it is not clear how to systematically improve the existing approaches, the currently most promising strategy is to understand and quantify the systematics and uncertainties of the models, for instance by varying assumptions about the colour topology, use of different hadronisation models etc.

Some of the implemented prescriptions have a potentially dangerous shortcoming in that they are infra-red and/or collinear sensitive. This is a point that could be improved upon.

Finally, there are alternative ideas (like pre-hadron formation) that should in principle be suitable for a MC implementation. Comparing new ideas to the traditional approaches could perhaps also lead to new insights.

## 4. Conclusions

There is strong motivation from both the theoretical and experimental side for studying jets in heavy ion collisions, although this implies a considerable increase of complexity

in the theoretical description. Among the conceptual issues raised by the transition from single-inclusive quantities to jets non-eikonal kinematics, multi gluon emission,  $k_{\perp}$ -broadening, jet-induced medium modifications and hadronisation are the most important ones. MC generators are powerful and versatile theory tools that allow to explore all these issues. They are designed to describe jets on the basis of multi-particle final states and allow to account dynamically for the interactions between the jet and the medium. However, as long as there is no general theory of jet quenching, also MC generators will have to rely on phenomenological models. It is to be expected that there will be considerable progress in the next years, driven by the jet data from RHIC and LHC and fruitful interaction between theorists and experimentalists.

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